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Estimating and Forecasting Volatility of Financial Time Series in Pakistan with GARCH-type Models

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Abstract

In this paper we compare the performance of different GARCH models such as GARCH, EGARCH, GJR and APARCH models, to characterize and forecast financial time series volatility in Pakistan. The comparison is carried out by comparing symmetric and asymmetric GARCH models with normal and fat-tailed distributions for the innovations, over short and long forecast horizons. The forecasts are evaluated according to a set of statistical loss functions. Daily data on the Karachi Stock Exchange (KSE) 100 index are analyzed. The empirical results demonstrate that the use of asymmetry in the GARCH models and the assumption of fat-tail distributions for the innovations improve the volatility forecasts. Overall, EGARCH fits the best while the GJR model, with both normal and non-normal innovations, seems to provide superior forecasting ability over short and long horizons.

Keywords: APARCH; EGARCH; Fat-tailed distribution; Forecast; Forecast horizon; GARCH; GJR; KSE 100; Volatility.

Introduction

Financial markets play a crucial role in any country's economy. Monetary policies are generally based on stock exchange indices, foreign exchange rates, price indices, inflation rates, interest rates, etc. Further it is generally assumed that the ultimate goal for monetary policy is price stability. Empirical studies have concluded that a large change in prices today tends to be followed by a larger change in the financial sector for which a time series study needs to be conducted. One has to carry a time

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series study of all such financial changes. Some well-known characteristics are common to many financial time series. Even a cursory look at data suggests that some time periods are riskier than others resulting in a variation in the expected values of the error terms. Moreover, these risky times are not scattered randomly across quarterly or annual data. Instead, there is a degree of autocorrelation in the riskiness of financial returns. Volatility clustering is often observed. Financial time series often exhibit leptokurtosis, meaning that the distribution of their returns is fat-tailed. Moreover, the so-called leverage effect refers to the fact that changes in stock prices tend to be negatively correlated with changes in volatility. The econometric challenge is to specify how the information is used to estimate and forecast the mean and variance of the return, conditional on the past information. Currently the most powerful known techniques used to estimate and predict the volatility on high frequency data belong to a family of generalized conditional autoregressive heteroskedastic (GARCH) models. The goal of such models is to provide a volatility measure like a standard deviation that can be used in financial decisions concerning risk analysis, portfolio selection and derivative pricing.

Primarily, time varying heteroskedasticity is modeled by Engle (1982). He proposed the autoregressive conditional heteroskedastic (ARCH) process that allows the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. Bollerslev (1986) extended his work and introduced the generalized autoregressive conditional heteroskedastic (GARCH) process. These models have been proved useful for modeling a variety of time series phenomena. However, both the models only control for the conditional heteroskedasticity, but they do not capture the so-called leverage effect. This led to the extension of nonlinear GARCH models e.g., the exponential GARCH (EGARCH) by Nelson (1991), GJR by Glosten, Jagannathan and Runkle (1993), the asymmetric power ARCH (APARCH) by Ding, Granger and Engle (1993), the Threshold GARCH of Zakoian (1994), the Quadratic GARCH (QARCH) by Santana (1995), etc. Although asymmetric models successfully capture the leverage effect, under the assumption of normal distribution of the innovation, they fail to capture the thick tail properties of financial time series. This has naturally led to the use of non-normal distributions, such as student-*t*, generalized error, normal Poisson, normal-lognormal, Bernoulli-normal, and skewed student-*t* distributions (see Peters, 2001 and the references therein).

The forecasting performance of GARCH models has been assessed many times e.g., Pagan and Schwert (1990), Brailsford and Faff (1996) and Loudon, Watt and Yadav (2000). On the other hand, comparing normal densities with non-normal ones, has also been studied in several times e.g., see Hsieh (1989), Baillie and Bollerslev (1989), Peters (2001) and Lambert and Laurent (2001).

The main goal of present study is to evaluate the performance of different GARCH models in terms of their ability to characterize and predict out-of-sample volatility of financial time series in Pakistan. For this purpose, we compare the forecasting ability of GARCH, EGARCH, GJR and APARCH models with normal, student-*t* and generalized error distribution (GED) innovations. The forecasting performance of such models is assessed through statistical loss functions. The estimates and forecasts are made on the KSE 100 index, because Pakistan's KSE 100 index is the best-performing stock market index in the world.

The plan of the paper is as follows: Section 2 discusses the models used in the study. Section 3 briefly describes the densities. In Section 4 we discuss forecast evaluation methods in terms of the statistical loss function to assess the forecast ability. All the empirical results and discussions are presented in Section 5 and some concluding remarks are made in Section 6.

2. Volatility Models

2.1. The GARCH Process

Let y_t denote the price index at time t =1, 2, ..., T and $r_t = ln(y_t / y_{t-1}) \times 100$ denote the rate of return from time t to t-1. Let ε_t be a real valued discrete - process and Ψ_t the information set (σ-field) of all information through time t. The ARMA(k, l)-GARCH (p, q) process is then defined as in (1)-(2)

$$r_t = \phi_0 + \sum_{i=1}^{k} \phi_{t-i} r_{t-i} + \sum_{j=1}^{l} \theta_{t-j} \varepsilon_{t-j} + \varepsilon_t \quad , \tag{1}$$

$$\varepsilon_t / \Psi_{t-1} \sim N\left(0, \sigma_t^2\right),$$

$$\sigma_t^2 = \gamma_0 + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \omega_j \sigma_{t-j}^2 \ , \tag{2}$$

where $p \geq 0$, $q > 0$, $\gamma_0 > 0$, $\gamma_i \geq 0$ for all $i = 1, 2, ..., q$ and $\omega_j \geq 0$ for $j = 0, 1, ..., p$. If p = 0 the GARCH (p, q) process reduces to the

ARCH (q) process and the conditional variance is simply a linear function of the past squared innovations only. If $p = q = 0$ then the GARCH process is simply white noise with constant unconditional variance. The GARCH process defined in (1) is stationary

$$\text{iff} \quad \sum_{i=1}^{q} \gamma_i + \sum_{j=1}^{p} `\omega_j < 1$$

Under the GARCH (p, q) process, the one-step-ahead volatility forecast may be given as

$$\hat{\sigma}^2_{T+1/T} = \hat{\gamma}_0 + \sum_{i=1}^{q} \hat{\gamma}_i \varepsilon^2_{T+1-i} + \sum_{j=1}^{p} \hat{\omega}_j \hat{\sigma}^2_{T+1-i} .$$

2.2. EGARCH Model

The exponential GARCH or EGARCH model involves the first introduction of an asymmetric effect on negative and positive shocks in an econometric model of volatility, by Nelson (1991). The specification for such a model is given as

$$ln\sigma_t^2 = \gamma_0 + \sum_{i=1}^{q} (\gamma_i |\eta_{t-i} - E(\eta_{t-i})| + \beta_i \eta_{t-i}) + \sum_{j=1}^{p} \omega_j \sigma^2_{t-j} ,$$

where $\eta_t = \dfrac{\varepsilon_t}{\sigma_t}$ is the standardized normal residual series.

The formulation in logarithm shares the usual positivity constraints on the parameters and also implies that the leverage effect is exponential rather than quadratic. The asymmetric effect is introduced by the non-linear function $\gamma_i |\eta_{t-i} - E(\eta_{t-i})| + \beta_i (\eta_{t-i})$ which is the function of both the magnitude and the sign of η_t. This specification has another advantage as compared to other asymmetric GARCH models; that is, it does not require any stationary constraints.

One step-ahead conditional variance forecast may be given as

$$\hat{\sigma}^2_{T+1|T} = \exp\left(\hat{\gamma}_0 + \sum_{i=1}^{q} (\hat{\gamma}_i |\eta_{T+1-i} - E(\eta_{T+1-i})| + \hat{\beta}_i \eta_{T+1-i}) + \sum_{j=1}^{p} \hat{\omega}_j \hat{\sigma}^2_{T_{T+1-i}} \right)$$

2.3. GJR Model

Gloston, Jagannathan and Runkle (1993) also consider the impact of good and bad news by introducing indicator function in the symmetric GARCH model

$$\sigma_t^2 = \gamma_0 + \sum_{i=1}^{q}(\gamma_i \varepsilon_{t-i}^2 + \beta_i d_{t-i}\varepsilon_{t-i}^2) + \sum_{j=1}^{p}\omega_j \sigma_{t-j}^2 ,$$

where d_t is the dummy variable and takes the value 0 when ε_t is positive and 1 when ε_t is negative. In other words the impact of ε_t^2 on the conditional variance is different when ε_t is positive or negative.

The one-step-ahead volatility forecast for the GJR model may be given as

$$\hat{\sigma}_{T+1|T}^2 = \hat{\gamma}_0 + \sum_{i=1}^{q}(\hat{\gamma}_i \varepsilon_{T+1-i}^2 + \hat{\beta}_i d_{T+1-i}\varepsilon_{T+1-i}^2) + \sum_{j=1}^{p}\hat{\omega}_j \hat{\sigma}_{T+1-j}^2 .$$

2.4. APARCH Model

The GARCH (*p, q*) model has been extended in various ways. Among the most interesting developments are the asymmetric power GARCH and APARCH (*p, q*) model (Ding, Granger and Engle, 1993), which allows to take account of both asymmetry and (possible) long memory property. The APARCH model can be expressed as

$$\sigma_t^\delta = \gamma_0 + \sum_{i=1}^{q}\gamma_i (|\eta_t| - \beta_i \eta_t)^\delta + \sum_{j=1}^{p}\omega_j \sigma_{t-j}^\delta ,$$

where $p \geq 0$, $q > 0$, $\gamma_0 > 0$, $\gamma_i \geq 0$, $-1 < \beta_i < 1$ for all $i = 1, 2, \ldots, q$, $\omega_j \geq 0$ for all $j = 1, 2, \ldots, p$ $\delta > 0$.

The covariance stationary condition for the model is

$$\sum_{i=1}^{q}\gamma_i (|\eta_t| - \beta_i \eta_t)^\delta + \sum_{j=1}^{p}\omega_j < 1 .$$

Ding, Granger and Engle (1993) found that the closer δ is to 1, the larger is the memory of the process. Equivalently, this model couples the flexibility of a varying exponent with an asymmetry coefficient. Moreover, the APARCH model includes seven other ARCH extensions as special cases (see, Peter, 2001, for more details).

One step-ahead volatility forecast may be given as

$$\hat{\sigma}^{\delta}_{T+1|T} = \hat{\gamma}_0 + \sum_{i=1}^{q} \hat{\gamma}_i (|\varepsilon_{T+1-i}| - \hat{\beta}_i \varepsilon_{T+1-i})^{\delta} + \sum_{j=1}^{p} \hat{\omega}_j \hat{\sigma}^{\delta}_{T+1-j} .$$

3. Densities

A normal density for innovation was assumed in the ARCH process introduced by Engle (1982) and Bollerslev (1986) who extended the ARCH process into GARCH. Although the normal distribution is widespread, it cannot effectively describe the thick tails of stock returns, due to excess kurtosis. Bollerslev and Wooldridge (1992) proposed quasi-maximum likelihood (QML) procedure which is robust to departures from normality. Although the QML estimator is consistent, it is inefficient for non-normality distributed data as the degree of inefficiency increases with the degree of departure from normality (Engle and Gonzalez-Rivera, 1991). This leads to the use of other distribution functions, such as the student-*t* by Bollerslev (1987) and generalized error distribution (GED) by Nelson (1991) to model tail thickness by a parameter, called degree of freedom.

3.1. Standardized Student-t Distribution

Bollerslev (1987) proposed the standardized student-*t* distribution with $\upsilon > 2$ degrees of freedom,

$$f(\eta_t) = \frac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\pi(\upsilon-2)}} \left(1 + \frac{\eta_t^2}{\upsilon-2}\right)^{-\frac{\upsilon+1}{2}} ,$$

where $\Gamma(.)$ is the gamma function. The degree of freedom represents the parameter to be estimated. The *t*-distribution is symmetric around zero and for $\upsilon > 4$ the conditional kurtosis equals $3(\upsilon-2)(\upsilon-4)^{-1}$, which exceeds the normal value of 3, but for $\upsilon \to \infty$ the density of standardized student-*t* distribution converges to the density function of the standardized normal distribution.

3.2. Generalized Error Distribution

Nelson (1991) suggested the use of the generalized error distribution (GED)

$$f(\eta_t) = \frac{\upsilon \exp(-0.5|\eta_t / \lambda|^{\upsilon})}{2^{(1+\frac{1}{\upsilon})}\Gamma(\upsilon^{-1})\lambda} \qquad \upsilon > 0,$$

where υ is the tail-thickness parameter and $\lambda \equiv \left[2^{(-2/\upsilon)}\Gamma(1/\upsilon)/\Gamma(3/\upsilon)\right]^{1/2}$. When $\upsilon = 2$, η_t is standard normally distributed. For $\upsilon < 2$, the distribution of η_t has thicker tails than the normal distribution (e.g., for $\upsilon = 1$, η_t has double exponential distribution) while for $\upsilon > 2$ the distribution of η_t has thinner tails than the normal distribution (e.g., for $\upsilon = \infty$, η_t has a uniform distribution on the interval $(-\sqrt{3}, \sqrt{3})$ (see Nelson, 1991). The conditional kurtosis is given by $(\Gamma(1/\upsilon)\Gamma(5/\upsilon))/(\Gamma(1/\upsilon))^2$.

Notice that the choice of a density has a particular impact on some models, for example in EGARCH the value of $E|\eta_t|$ depends on the density function for the standard normal distribution

$$E(\eta_{t-i}) = \sqrt{\frac{2}{\pi}},$$

for student-*t* distribution

$$E(|\eta_{t-i}|) = \frac{2\Gamma(\frac{1+\upsilon}{2})^2\sqrt{(\upsilon-2)}}{1+\sqrt{\pi}(\upsilon-1)\Gamma(\upsilon/2)},$$

for GED

$$E(|\eta_{t-i}|) = \lambda 2^{1/\upsilon}\frac{\Gamma(2/\upsilon)}{\Gamma(1/\upsilon)}.$$

4. Forecast Evaluation Methods

The comparison of forecasting performance of GARCH models requires the actual volatility denoted by σ_t^2. As such, it provides the natural benchmark for forecast evaluation purposes. A common model-free indicator of volatility is the daily squared return. However, one can obtain a more accurate measure by following an idea proposed by Merton (1980) and Schwert (1989) and formalized by Andersen and Bollerslev (1998). They argued that the single squared change is a noisy indicator for the latent volatility in the period, because the idiosyncratic component of a single change is large. The noise is reduced by taking the sum of all squared intra-period changes, and the smaller the sub-period, the larger the noise reduction. Since the highest frequency available to us is daily data, this idea results in the use of the daily squared return $\sigma_t^2 = r_t^2$ as actual volatility.

We have summed the daily realized volatility over the *k*-days to obtain the volatility at *k*-step-ahead (for k >1) i.e. $\sigma_{T+k}^2 = \sum_{j=1}^{k} \sigma_{T+j}^2$.

Similarly, *k*-step-ahead volatility forecast $\hat{\sigma}_{T+k|T}^2$ is the aggregated sum of the forecasts made at time T i.e. $\hat{\sigma}_{T+k|T}^2 = \sum_{j=1}^{k} \hat{\sigma}_{T+j|T}^2$.

The evaluation of forecast ability of competing volatility models is not an easy task, as pointed by Bollerslev, Engle and Nelson (1994), and Lopez (2001), and there does not exist an exceptional measure of selecting the best model. Hansen, Lunde and Nason (2003b) applied the Model Confidence Set (MCS) procedure of Hansen, Lunde and Nason (2003a) to a set of volatility models in order to pick the 'best' forecasting model, amongst case volatility models. As in this approach, the performance of a forecast may be evaluated by using an out-of-sample evaluation under a loss function specified by the user. But like many researchers (e.g., Peter, 2001 and Marcucci, 2005), this paper simply uses different statistical loss functions, available in literature for volatility forecast evaluation. These loss functions will be used as diagnostic tools on the forecasting model.

To assess the forecast ability of different models, the paper also uses some statistical loss functions that have different interpretations. These are given as:

1. $$MSE1 = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\sigma_{t+1} - \hat{\sigma}_{t+1|T}\right)^2 .$$

2. $$MSE2 = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\sigma_{t+1}^2 - \hat{\sigma}_{t+1|T}^2\right)^2 .$$

3. $$MAE1 = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\left|\sigma_{t+1}^2 - \hat{\sigma}_{t+1|T}^2\right|\right).$$

4. $$MAE2 = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\left|\sigma_{t+1} - \hat{\sigma}_{t+1|T}\right|\right).$$

5. $$MAPE = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\left|\frac{\sigma_{t+1}^2 - \hat{\sigma}_{t+1|T}^2}{\sigma_{t+1}^2}\right|\right).$$

6. $$TIC = \frac{\sqrt{\frac{1}{h+1}\sum_{t=T}^{T+h}\left(\sigma_{t+1}^2 - \hat{\sigma}_{t+1|T}^2\right)^2}}{\sqrt{\frac{1}{h+1}\sum_{t=T}^{T+h}\left(\sigma_{t+1}^2\right)} + \sqrt{\frac{1}{h+1}\sum_{t=T}^{T+h}\left(\hat{\sigma}_{t+1|T}^2\right)}} .$$

7. Mincer-Zarnowitz R^2.

8. $$R^2 LOG = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\log(\sigma_{t+1}^2 / \hat{\sigma}_{t+1|T}^2)\right)^2 .$$

9. $$HMSE = \frac{1}{h+1}\sum_{t=T}^{T+h}\left(\sigma_{t+1}^2 / \hat{\sigma}_{t+1|T}^2 - 1\right)^2 .$$

In the above cases h is the forecast horizon.

The first two measures are the mean square error (MSE). These forecast error statistics depend on the scale of the dependent variable. The criteria (3), (4) and (5) are the mean absolute error (MAE) and mean absolute percentage errors (MAPE), respectively. The MSE's are more sensitive to outliers than MAE's. The measure in (6) is the Theil inequality coefficient (TIC) which is scale invariant. It always lies between zero and one, where zero indicates a perfect fit. The loss function in (7) is computed in Mincer-Zarnowitz regressions (Mincer-Zarnowitz 1969), by regressing the actual variance σ^2_{T+k} on the constant and forecasted variance $\hat{\sigma}^2_{T+k|T}$,

$$\sigma^2_{T+k} = a + b\hat{\sigma}^2_{T+k|T} + v_{T+k}.$$

The statistic R^2 from this regression provides the proportion of variance explained by the forecast i.e. the higher the R^2, better the forecasts. The R^2 LOG, named by Pagan and Schwert (1990) as the logarithmic loss function, penalizes volatility forecasts asymmetrically in low and high volatility periods. The loss function in (9) is the k-adjusted MSE (HMSE), proposed by Bollerslev and Ghysels (1996).

5. Empirical Results and Discussions

5.1. Data and Methodology

In this section, we describe the data and our methodology. The whole sample consists of the KSE 100 index of Pakistan closing prices from January 1, 2002 to August 31, 2006, for a total of 1218 observations. The estimation process is run using four years of data (2002-2005) while the remaining eight months (January 1, 2006 to August 31, 2006) data are used for the evaluation of the out of sample forecast performance. The indices prices are transformed into their rates of returns.

First of all, the statistical properties of returns are assessed through means of coefficients of skewness and kurtosis, Jarque-Bera test of normality, ARCH LM test and Ljung-Box test on the squared residuals to check the presence of typical stylized facts.

Table-5.1: Descriptive Statistics of r_t

Mean	St .Dev	Min.	Max.	Skewness	Kurtosis	Jarque-Bera test	LM(10)	$Q^2(10)$
0.1671	1.5887	-7.7408	11.6000	-0.1958	6.9837	811.8576	225.1666	536.2900

Table-5.1, represents the descriptive statistics of r_t. The Jarque-Bera statistic is high due to excess kurtosis and negative skewness, indicating the non-normality of the distribution. Moreover, LM (10) statistics is the ARCH-LM test proposed by Engle (1982), $Q^2(10)$ is the Ljung-Box test statistics on the squared residuals up to lag of 10. Under the null of no serial correlation, the high values for both the statistics indicate the presence of ARCH effect in the conditional variance.

For the identification of the mean model, we have followed the Box-Jenkins methodology. A number of tentative models with increasing ARMA orders and increasing GARCH orders have been estimated. Appropriate models are identified using autocorrelation function (ACF), partial autocorrelation function (PACF) and Ljung-Box statistics of the standardized residuals and the squared standardized residuals and ARCH-LM test. Through this exercise, a GARCH (1, 1) process is found to be the best model for conditional variance. The final model amongst the models, satisfying the diagnostics is selected on the basis of Akiake information criterion (AIC) and Schwarz's Bayesian information criterion (BIC) given in the Appendix. The selected model is given as

$$r_t = \phi_0 + \phi_9 r_{t-9} + \varepsilon_t .$$

Table-1, presents the estimation results for the parameters for the mean model, GARCH, GJR, EGARCH and APARCH models with three distributions: normal, student-*t* and GED. Asymptotic k-consistent standard errors are given in parentheses. To estimate and forecast volatility, we use the popular software, EViews 5.0

Table-1: Estimates of Symmetric and Asymmetric GARCH Models with Different Conditional Distributions

	GARCH-N	GARCH-St	GARCH -GED	GJR-N	GJR - St	GJR - GED	EGARCH-N	EGARCH - St	EGARCH -GED	APARCH-N	APARCH -St	APRCH -GED
ϕ_0	0.2366*	0.2661*	0.2297*	0.2288*	0.2586*	0.2245*	0.2311*	0.2553*	0.2227*	0.2327*	0.2552*	0.2242*
	(0.0350)	(0.0314)	(0.0298)	(0.0363)	(0.0319)	(0.0303)	(0.0398)	(0.0307)	(0.0291)	(0.0308)	(0.0311)	(0.0294)
ϕ_9	0.0232	0.0399	0.0270	0.0258	0.0412	0.0280	0.0223	0.0393	0.0269	0.0229	0.0397	0.0274
	(0.0284)	(0.0270)	(0.0266)	(0.0319)	(0.0264)	(0.0261)	(0.0280)	(0.0261)	(0.0259)	(0.0315)	(0.0262)	(0.0258)
γ_0	0.1131*	0.0956*	0.0995*	0.1286*	0.1085*	0.1145*	0.2350*	0.2914*	0.2656*	0.1063*	0.0875*	0.0980*
	(0.0314)	(0.0291)	(0.0300)	(0.0228)	(0.0315)	(0.0331)	(0.0483)	(0.0400)	(0.0378)	(0.0216)	(0.0272)	(0.0305)
γ_1	0.2081*	0.2652*	0.2289*	0.1827*	0.2217*	0.1950*	0.3643*	0.4390*	0.3999*	0.2117*	0.2592*	0.2352*
	(0.0455)	(0.0540)	(0.0445)	(0.0305)	(0.0582)	(0.0514)	(0.0648)	(0.0636)	(0.0582)	(0.0231)	(0.0467)	(0.0424)
ω_1	0.7502*	0.7302*	0.7416*	0.7343*	0.7169*	0.7248*	0.9274*	0.9379*	0.9301*	0.7655*	0.7514*	0.7544*
	(0.0423)	(0.0394)	(0.0403)	(0.0273)	(0.0412)	(0.0430)	(0.0177)	(0.0181)	(0.0201)	(0.0281)	(0.0403)	(0.0435)
β_1	-	-	-	0.0652*	0.1001	0.0866	-0.042*	-0.0454	-0.0487	0.1319*	0.1179	0.1364
				(0.0350)	(0.0750)	(0.0659)	(0.0409)	(0.0344)	(0.0323)	(0.0639)	(0.0845)	(0.0935)
δ	-	-	-	-	-	-	-	-	-	1.2087*	1.2529*	1.2244*
										(0.2603)	(0.3587)	(0.3855)
υ		4.5760*	1.2185*		4.5446*	1.2182*		4.6618*	1.2285*		4.6722*	1.2265*
		(0.6996)	(0.0647)		(0.6963)	(0.0656)		(0.7260)	(0.0666)		(0.7181)	(0.0656)
$Q(12)$	15.08	15.404	14.531	14.98	15.339	14.37	15.381	15.725	14.724	15.155	15.518	14.643
$Q^2(12)$	5.9887	6.1195	6.3398	5.1828	5.4006	5.71	6.1556	6.0281	6.4493	5.7019	5.4891	5.7759

Note: Each GARCH model has been estimated with a normal (N), student-*t* (St) and a GED distribution. Asymptotic heteroscedasticity-consistent standard errors are given in parentheses. "*" indicates the significance of the estimates. *$Q(12$ and $Q^2(12$ are* the Ljung-Box statistics at lag 12.

Regarding the conditional mean, $\hat{\phi}_0$ is highly significant for all the models. However, $\hat{\phi}_9$ is non-significant, although we do not drop this parameter because by doing so, the ACF of the standardized residuals becomes significant at lag 9. Moreover, as our main focus is on the forecasts of volatility and by dropping this parameter, the forecast's accuracy reduces. The conditional variance estimates show that all the parameters are highly significant except asymmetric parameters in the cases of student-*t* and GED distributions. In addition, for the student-*t* distribution, the values of shape parameter υ for GARCH, EGARCH, GJR and APARCH clearly indicate the typical fat-tail behavior of financial returns. Moreover, for the GED, the estimates clearly suggest that the conditional distribution has fatter tails than the normal distribution, since the shape parameters for GARCH, EGARCH, GJR and APARCH have values that significantly between 1 and 2 indicating the conditional distribution of KSE 100 index is indeed fat-tailed. Ljung Box statistics at lag 12, $Q(12)$ and $Q^2(12)$ on the standardized residuals and the squared standardized residuals respectively, are non-significant indicating that all these models adequately described the dynamics of the series.

Table-2: In-sample Measures of Goodness-of-fit (Models Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log (L)	Rank	AIC	Rank	BIC	Rank	Sum Rank	Rank
Normal-Distribution																								
GARCH	1.1478	4	34.0721	4	2.4680	4	2.3532	4	0.5585	3	0.8226	4	1.3870	4	4.6662	3	-1722.87	4	3.3389	3	3.3628	2	39	4
GJR	1.1351	3	33.1513	3	2.4500	3	2.3354	3	0.5513	1	0.8197	3	1.3849	3	4.6651	2	-1722.06	3	3.3393	4	3.3679	3	31	3
EGARCH	1.0940	1	31.8213	1	2.3948	1	2.2862	1	0.5697	4	0.8135	1	1.3830	2	4.6216	1	-1717.76	1	3.3309	1	3.3596	1	15	1
APARCH	1.1143	2	32.3411	2	2.4273	2	2.3145	2	0.5530	2	0.8157	2	1.3830	1	4.7328	4	-1718.6	2	3.3345	2	3.3679	4	25	2
Student-t Distribution																								
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log(L)	Rank	AIC	Rank	BIC	Rank	sum	sum Rank
GARCH	1.2147	4	35.0137	4	2.5733	4	2.4499	4	0.5360	3	0.8406	4	1.3972	4	4.8092	1	-1673.23	4	3.2449	3	3.2736	1	36	4
GJR	1.2056	3	34.0130	3	2.5625	3	2.4379	3	0.5260	2	0.8393	3	1.3959	3	4.8238	2	-1672.37	3	3.2452	4	3.2786	3	32	3
EGARCH	1.1486	1	32.0584	1	2.4855	1	2.3694	1	0.5378	4	0.8301	1	1.3921	2	4.9137	3	-1670.77	2	3.2421	1	3.2755	2	19	1
APARCH	1.1816	2	33.1271	2	2.5260	2	2.4040	2	0.5258	1	0.8316	2	1.3901	1	5.0367	4	-1670.21	1	3.2429	2	3.2811	4	23	2
Generalized Errors Distribution																								
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log (L)	Rank	AIC	Rank	BIC	Rank	sum	sum Rank
GARCH	1.1609	4	33.7022	4	2.4880	4	2.3724	4	0.5510	3	0.8223	4	1.3766	4	5.0191	2	-1679.29	4	3.2566	3	3.2853	1	37	4
GJR	1.1493	3	32.9369	3	2.4701	3	2.3540	3	0.5411	2	0.8195	3	1.3742	3	5.0382	3	-1678.47	3	3.257	4	3.2904	3	33	3
EGARCH	1.1015	1	31.7533	1	2.4091	1	2.3004	1	0.5576	4	0.8128	1	1.3732	2	5.0126	1	-1676.35	2	3.2529	1	3.2863	2	17	1
APARCH	1.1293	2	32.1318	2	2.4483	2	2.3343	2	0.5409	1	0.8153	2	1.3726	1	5.1609	4	-1676.32	1	3.2547	2	3.2929	4	23	2

Note: MSE1, MSE2, MAE1, MAPE, TIC, MAE2, R2LOG and HMSE are the statistical loss functions given in Section 4. Log (L) is the log-likelihood value, AIC is the Akiake information criteria, BIC is the Schwarz' s information criteria and Sum Rank is the sum of the ranks of the individual loss functions. Final Rank is the Rank of Sum Rank.

Table-2 shows the model comparison in terms of measures of goodness of fit. The results demonstrate that the performance of asymmetric GARCH models with all the three distributions justified the use of asymmetric GARCH models to estimate the series as highlighted by the values of the log-likelihood. According to AIC, EGARCH perform the best in all the three cases. According to the statistical loss functions considered in this study, the EGARCH model with normal and non-normal innovations fits the best, since the sum of the ranks is the smallest. The second best model is the APARCH. However, the performance of GARCH model with all the three distributions is poorest, as the sum of the ranks of all the measures is highest in each case.

Table-3 : In-sample Measures of Goodness-of-fit (Distribution Comparison)

Distribution	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log(L)	Rank	AIC	Rank	BIC	Rank	Sum Rank	Rank
GARCH																								
Normal	1.1478	1	34.0721	2	2.4680	1	2.3532	1	0.5585	3	0.8226	2	1.3870	2	4.6662	1	-1722.87	3	3.3389	3	3.3628	3	22	2
Student-*t*	1.2147	3	35.0137	3	2.5733	3	2.4499	3	0.5360	1	0.8406	3	1.3972	3	4.8092	2	-1673.23	1	3.2449	1	3.2736	1	24	3
GED	1.1609	2	33.7022	1	2.4880	2	2.3724	2	0.5510	2	0.8223	1	1.3766	1	5.0191	3	-1679.29	2	3.2566	2	3.2853	2	20	1
GJR																								
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log(L)	Rank	AIC	Rank	BIC	Rank	Sum Rank	Rank
Normal	1.1351	1	33.1513	2	2.4500	1	2.3354	1	0.5513	3	0.8197	2	1.3849	2	4.6651	1	-1722.06	3	3.3393	3	3.3679	3	22	2
Student-*t*	1.2056	3	34.0130	3	2.5625	3	2.4379	3	0.5260	1	0.8393	3	1.3959	3	4.8238	2	-1672.37	1	3.2452	1	3.2786	1	24	3
GED	1.1493	2	32.9369	1	2.4701	2	2.3540	2	0.5411	2	0.8195	1	1.3742	1	5.0382	3	-1678.47	2	3.257	2	3.2904	2	20	1
EGARCH																								
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log(L)	Rank	AIC	Rank	BIC	Rank	Sum Rank	Rank
Normal	1.0940	1	31.8213	2	2.3948	1	2.2862	1	0.5697	3	0.8135	2	1.3830	2	4.6216	1	-1717.76	3	3.3309	3	3.3596	3	22	2
Student-*t*	1.1486	3	32.0584	3	2.4855	3	2.3694	3	0.5378	1	0.8301	3	1.3921	3	4.9137	2	-1670.77	1	3.2421	1	3.2755	1	24	3
GED	1.1015	2	31.7533	1	2.4091	2	2.3004	2	0.5576	2	0.8128	1	1.3732	1	5.0126	3	-1676.35	2	3.2529	2	3.2863	2	20	1
APARCH																								
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log(L)	Rank	AIC	Rank	BIC	Rank	Sum Rank	Rank
Normal	1.1143	1	32.3411	2	2.4273	1	2.3145	1	0.5530	3	0.8157	2	1.3830	2	4.7328	1	-1718.6	3	3.3345	3	3.3679	3	22	2
Student-*t*	1.1816	3	33.1271	3	2.5260	3	2.4040	3	0.5258	1	0.8316	3	1.3901	3	5.0367	2	-1670.21	1	3.2429	1	3.2811	1	24	3
GED	1.1293	2	32.1318	1	2.4483	2	2.3343	2	0.5409	2	0.8153	1	1.3726	1	5.1609	3	-1676.32	2	3.2547	2	3.2929	2	20	1

Table-3 shows the distribution comparison in terms of measures that calculate goodness of fit. The results show that the overall comparison is difficult. According to the log-likelihood, AIC and BIC, the competing models fit the best with fat-tailed distributions and prominent student-*t*, while the symmetric and asymmetric GARCH models with normal distribution perform the poorest. According to other measures, all the competing models with student-*t* innovations perform the poorest. Overall, on the basis of all the measures, all the competing models fit best on the series with GED innovations.

Table-4: In-sample Measures of Goodness-of-fit (Overall Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	Log(L)	Rank	AIC	Rank	BIC	Rank	Sum Rank	Rank
GARCH-N	1.1478	6	34.0721	11	2.4680	6	2.3532	6	0.5585	11	0.8226	8	1.3870	8	4.6662	3	-1722.87	12	3.3389	11	3.3628	10	92	12
GARCH- St	1.2147	12	35.0137	12	2.5733	12	2.4499	12	0.5360	3	0.8406	12	1.3972	12	4.8092	5	-1673.23	4	3.2449	3	3.2736	1	88	11
GARCH-GED	1.1609	9	33.7022	9	2.4880	9	2.3724	9	0.5510	7	0.8223	7	1.3766	4	5.0191	9	-1679.29	8	3.2566	7	3.2853	5	83	9.5
GJR-N	1.1351	5	33.1513	8	2.4500	5	2.3354	5	0.5513	8	0.8197	6	1.3849	7	4.6651	2	-1722.06	11	3.3393	12	3.3679	11	80	8
GJR - St	1.2056	11	34.0130	10	2.5625	11	2.4379	11	0.5260	2	0.8393	11	1.3959	11	4.8238	6	-1672.37	3	3.2452	4	3.2786	3	83	9.5
GJR - GED	1.1493	8	32.9369	6	2.4701	7	2.3540	7	0.5411	6	0.8195	5	1.3742	3	5.0382	11	-1678.47	7	3.2570	8	3.2904	7	75	7
EGARCH-N	1.0940	1	31.8213	2	2.3948	1	2.2862	1	0.5697	12	0.8135	2	1.3830	6	4.6216	1	-1717.76	9	3.3309	9	3.3596	9	53	2
EGARCH-St	1.1486	7	32.0584	3	2.4855	8	2.3694	8	0.5378	4	0.8301	9	1.3921	10	4.9137	7	-1670.77	2	3.2421	1	3.2755	2	61	4
EGARCH-GED	1.1015	2	31.7533	1	2.4091	2	2.3004	2	0.5576	10	0.8128	1	1.3732	2	5.0126	8	-1676.35	6	3.2529	5	3.2863	6	45	1
APARCH-N	1.1143	3	32.3411	5	2.4273	3	2.3145	3	0.5530	9	0.8157	4	1.3830	5	4.7328	4	-1718.6	10	3.3345	10	3.3679	12	68	5
APARCH-St	1.1816	10	33.1271	7	2.5260	10	2.4040	10	0.5258	1	0.8316	10	1.3901	9	5.0367	10	-1670.21	1	3.2429	2	3.2811	4	74	6
APARCH-GED	1.1293	4	32.1318	4	2.4483	4	2.3343	4	0.5409	5	0.8153	3	1.3726	1	5.1609	12	-1676.32	5	3.2547	6	3.2929	8	56	3

Table-4 shows the overall in-sample measures of goodness of fit. The overall comparison shows that the largest log-likelihood is given by the APARCH model with student-*t* innovations, while AIC indicates that the best model is EGARCH with student-*t* innovations. Overall, the sum of the ranks of all statistical loss functions show that the EGARCH models with GED and normal innovations respectively fit the best followed by the EGARCH and APARCH models while the performance of the GARCH model is the poorest.

5.2. Forecast Evaluation

The main goal of our study is to compare the forecasting ability of different GARCH models. Such a comparison has been carried out by comparing the volatility forecasts at one-, five- ten-, fifteen- and twenty-steps-ahead. Forecasting ability of competing GARCH models is reported by ranking according to the statistical loss functions given in section 4 through Table-5 to Table-11. We have compared the results in terms of model comparisons and distribution comparisons at all the one-, five- ten-, fifteen- and twenty-steps-ahead forecast horizons. But the scope of the present paper has been limited to the case of ten-steps, as the rest of the cases follow a similar pattern. However, the total comparison is given for all the forecast horizons. Finally the best performing model is selected by ranking the sum of the ranks of the individual loss functions.

Table-5: One-step-ahead Volatility Forecast (Overall Comparison)

Model	*MSE1*	*Rank*	*MSE2*	*Rank*	*MAE1*	*Rank*	*MAPE*	*Rank*	*TIC*	*Rank*	*MAE2*	*Rank*	*R2LOG*	*Rank*	*HMSE*	*Rank*	$\bar{R}$	*Rank*	*Sum Rank*	*Rank*
GARCH-N	1.3911	6	26.2033	9	3.4340	9	3.3396	9	0.4290	11	0.9741	9	1.2592	6	2.8967	11	26.80%	12	82	10
GARCH- St	1.4621	11	26.6177	11	3.4721	12	3.3720	12	0.4062	4	0.9832	12	1.2850	11	2.7673	7	27.30%	10	90	11
GARCH- GED	1.4095	9	26.2764	10	3.4430	10	3.3486	10	0.4211	9	0.9751	10	1.2629	8	3.0169	12	26.90%	11	89	12
GJR-N	1.3613	3	25.1544	2	3.3609	2	3.2656	2	0.4141	7	0.9590	2	1.2505	1	2.7746	8	29.70%	3	30	2
GJR - St	1.4438	10	25.7453	7	3.4050	7	3.3032	7	0.3898	1	0.9694	7	1.2791	10	2.6138	4	30.50%	1	54	5
GJR - GED	1.3810	5	25.1091	1	3.3526	1	3.2570	1	0.4024	3	0.9570	1	1.2547	4	2.8549	10	30.20%	2	28	1
EGARCH-N	1.3424	1	25.6853	5	3.3718	4	3.2764	4	0.4401	12	0.9626	4	1.2509	2	2.5900	2	28.90%	6	40	4
EGARCH- St	1.3985	7	25.4276	4	3.4059	8	3.3051	8	0.4107	6	0.9714	8	1.2753	9	2.4645	1	29.30%	4	55	6
EGARCH- GED	1.3482	2	25.3367	3	3.3624	3	3.2668	3	0.4275	10	0.9600	3	1.2515	3	2.6647	5	29.00%	5	37	3
APARCH-N	1.3788	4	25.6954	6	3.3876	5	3.2917	5	0.4203	8	0.9645	6	1.2584	5	2.6851	6	28.20%	9	54	7
APARCH-St	1.4690	12	26.6698	12	3.4525	11	3.3512	11	0.3987	2	0.9779	11	1.2864	12	2.5943	3	28.30%	8	82	9
APARCH-GED	1.4026	8	25.8091	8	3.3917	6	3.2954	6	0.4090	5	0.9644	5	1.2624	7	2.7888	9	28.50%	7	61	8

5.3.1. One-step-ahead Forecast Evaluation

Table-5 shows the forecast evaluation at one step ahead. The model comparison recommends that asymmetric GARCH models perform the best for all the three distributions made obvious by final ranks. For all the three distributions, the pattern of the ranks of the final ranks (the ranks of the sum of the individual loss functions) is (4, 1, 2, 3), for GARCH, GJR, EGARCH and APARCH respectively. This indicates that the first best model is the GJR and the second best model is EGARCH. APARCH provide less satisfactory results while symmetric GARCH, clearly, gives the poorest forecasts.

The comparison between densities is harder because results vary across models. The symmetric GARCH and APARCH show the pattern of the ranks of the final ranks as (1, 3, 2), for normal, student-*t* and GED respectively, indicating the best results are obtained with normal innovations. While GJR and EGARCH gives the final ranks as (2, 3, 1) for normal, student-*t* and GED respectively revealing the best results with GED innovations. At one-step-ahead, the forecasting ability of all the competing models with student-*t* innovation is the poorest.

The overall comparison of the forecasting performance of the competing models shows that GJR model with GED innovations seems to perform the best.

Table-6: Five-step-ahead Volatility Forecast (Overall Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	$\bar{R}$	Rank	Sum Rank	Rank
GARCH-N	1.32432	12	156.3150	11	8.2034	12	0.7067	11	0.2376	12	0.9095	12	0.0764	12	0.3364	11	67.20%	12	105	12
GARCH- St	1.27149	9	143.2830	7	8.0344	10	0.7199	12	0.2316	9	0.8837	8	0.0743	11	0.2586	5	68.90%	10	81	9
GARCH- GED	1.28468	10	149.6940	10	8.0797	11	0.6926	9	0.2337	10	0.8918	11	0.0737	9	0.3111	9	67.70%	11	90	11
GJR-N	1.15814	3	134.4730	4	7.6944	3	0.6692	4	0.2220	3	0.8560	3	0.0697	5	0.2860	6	71.70%	4	35	3
GJR - St	1.13908	2	124.1580	1	7.5075	2	0.6848	7	0.2188	2	0.8315	2	0.0687	2	0.2237	1	73.70%	1	20	2
GJR - GED	1.10302	1	124.5740	2	7.4853	1	0.6509	1	0.2162	1	0.8301	1	0.0663	1	0.2572	4	72.90%	2	14	1
EGARCH-N	1.29718	11	161.8200	12	8.0021	9	0.6813	5	0.2358	11	0.8899	10	0.0739	10	0.3382	12	70.50%	6	86	10
EGARCH- St	1.16652	4	132.8610	3	7.7133	4	0.6906	8	0.2224	4	0.8563	4	0.0709	6	0.2550	2	71.80%	3	38	4
EGARCH- GED	1.19415	6	145.8940	9	7.7513	5	0.6568	2	0.2259	6	0.8585	5	0.0693	4	0.2991	8	71.70%	4	49	6
APARCH-N	1.24599	7	145.5180	8	7.9715	8	0.6817	6	0.2303	8	0.8854	9	0.0726	8	0.3209	10	69.30%	9	73	8
APARCH-St	1.25041	8	139.6170	6	7.9643	7	0.7037	10	0.2293	7	0.8747	7	0.0722	7	0.2558	3	70.30%	7.5	62.5	7
APARCH-GED	1.19267	5	136.2520	5	7.8351	6	0.6653	3	0.2248	5	0.8647	6	0.0692	3	0.2894	7	70.30%	7.5	47.5	5

5.3.2. Five- step-ahead Forecast Evaluation

Table-6 shows the forecast comparison at five-steps-ahead. The model comparison gives a pattern similar to the final ranks at one-step-ahead forecast horizon. So, the use of asymmetric GARCH model versus the symmetric GARCH is strongly recommended. For all three distributions the first best model is again the GJR and the second best model is EGARCH. APARCH provides less satisfactory results while symmetric GARCH clearly gives the poorest forecasts.

The comparison between densities led to the use of non-normal densities since all the competing models give better forecasts with fat-tail distributions. The symmetric GARCH and EGARCH show the pattern of the ranks of the final ranks as (3, 1, 2), for normal, student-t and GED respectively, indicating the best results lie with student-t innovations. Moreover, GJR and APARCH give the final ranks as (2, 3, 1) for normal, student-t and GED respectively revealing the best results with GED innovations. At five-steps-ahead the forecasting ability of all the competing models with normal innovation is poorest.

Overall results illustrate that the GJR model with GED is again the most successful model to forecast the volatility of KSE 100 at five steps-ahead.

Table-7: Ten-step-ahead Volatility Forecast (Model Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	R^2	Rank	Sum Rank	Rank
Normal-Distribution																				
GARCH	1.1690	3	215.7860	3	10.3088	3	0.4199	4	0.1618	3	0.8555	3	0.0367	4	0.1516	3	86.20%	4	30	3
GJR	0.9685	1	173.7340	1	9.2787	1	0.3878	1	0.1471	1	0.7787	1	0.0321	1	0.1286	1	88.70%	1	9	1
EGARCH	1.2628	4	275.1420	4	11.3735	4	0.4189	3	0.1685	4	0.9046	4	0.0361	3	0.1579	4	87.40%	2	32	4
APARCH	1.0875	2	199.9680	2	9.9860	2	0.4059	2	0.1559	2	0.8289	2	0.0345	2	0.1435	2	86.90%	3	19	2
Student-t Distribution																				
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	R^2	Rank	Sum Rank	Rank
GARCH	1.0265	3	168.1730	2	9.3143	3	0.4090	4	0.1510	3	0.7794	3	0.0339	4	0.1143	3	87.80%	4	29	3
GJR	0.9204	1	148.6420	1	8.7418	1	0.3883	1	0.1428	1	0.7364	1	0.0308	1	0.1026	1	90.10%	1	9	1
EGARCH	0.9617	2	169.3140	3	9.2365	2	0.3979	2	0.1464	2	0.7709	2	0.0323	2	0.1132	2	88.50%	2	19	2
APARCH	1.0349	4	172.8760	4	9.5142	4	0.4075	3	0.1514	4	0.7880	4	0.0333	3	0.1144	4	88.10%	3	33	4
Generalized Errors Distribution																				
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	R^2	Rank	Sum Rank	Rank
GARCH	1.0763	4	190.0480	3	9.5939	3	0.3991	4	0.1550	3	0.8026	3	0.0342	4	0.1362	4	86.80%	4	32	4
GJR	0.8729	1	146.1900	1	8.6625	1	0.3671	1	0.1395	1	0.7304	1	0.0293	1	0.1130	1	89.50%	1	9	1
EGARCH	1.0732	3	223.6930	4	10.2588	4	0.3893	3	0.1551	4	0.8247	4	0.0321	3	0.1341	3	88.80%	2	30	3
APARCH	0.9780	2	168.7410	2	9.2602	2	0.3843	2	0.1476	2	0.7749	2	0.0316	2	0.1258	2	87.80%	3	19	2

Table-8: Ten-step-ahead Volatility Forecast (Distribution Comparison)

Distribution	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	$\bar{R}^2$	Rank	Sum Rank	Rank
GARCH																				
Normal	1.1690	3	215.7860	3	10.3088	3	0.4199	3	0.1618	3	0.8555	3	0.0367	3	0.1516	3	86.20%	3	27	3
Student-*t*	1.0265	1	168.1730	1	9.3143	1	0.4090	2	0.1510	1	0.7794	1	0.0339	1	0.1143	1	87.80%	1	10	1
GED	1.0763	2	190.0480	2	9.5939	2	0.3991	1	0.1550	2	0.8026	2	0.0342	2	0.1362	2	86.80%	2	17	2
GJR																				
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	$\bar{R}^2$	Rank	Sum Rank	Rank
Normal	0.9685	3	173.7340	3	9.2787	3	0.3878	2	0.1471	3	0.7787	3	0.0321	3	0.1286	3	88.70%	3	26	3
Student-*t*	0.9204	2	148.6420	2	8.7418	2	0.3883	3	0.1428	2	0.7364	2	0.0308	2	0.1026	1	90.10%	1	17	2
GED	0.8729	1	146.1900	1	8.6625	1	0.3671	1	0.1395	1	0.7304	1	0.0293	1	0.1130	2	89.50%	2	11	1
EGARCH																				
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	$\bar{R}^2$	Rank	Sum Rank	Rank
Normal	1.2628	3	275.1420	3	11.3735	3	0.4189	3	0.1685	3	0.9046	3	0.0361	3	0.1579	3	87.40%	3	27	3
Student-*t*	0.9617	1	169.3140	1	9.2365	1	0.3979	2	0.1464	1	0.7709	1	0.0323	2	0.1132	1	88.50%	2	12	1
GED	1.0732	2	223.6930	2	10.2588	2	0.3893	1	0.1551	2	0.8247	2	0.0321	1	0.1341	2	88.80%	1	15	2
APARCH																				
	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	$\bar{R}^2$	Rank	Sum Rank	Rank
Normal	1.0875	3	199.9680	3	9.9860	3	0.4059	2	0.1559	3	0.8289	3	0.0345	3	0.1435	3	86.90%	3	26	3
Student-*t*	1.0349	2	172.8760	2	9.5142	2	0.4075	3	0.1514	2	0.7880	2	0.0333	2	0.1144	1	88.10%	1	17	2
GED	0.9780	1	168.7410	1	9.2602	1	0.3843	1	0.1476	1	0.7749	1	0.0316	1	0.1258	2	87.80%	2.0	11	1

Table-9: Ten-step-ahead Volatility Forecast (Overall Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	R^2	Rank	Sum Rank	Rank
GARCH-N	1.1690	11	215.7860	10	10.3088	11	0.4199	12	0.1618	11	0.8555	11	0.0367	12	0.1516	11	86.20%	12	101	11
GARCH- St	1.0265	6	168.1730	3	9.3143	6	0.4090	10	0.1510	6	0.7794	6	0.0339	8	0.1143	4	87.80%	7.5	56.5	6
GARCH- GED	1.0763	9	190.0480	8	9.5939	8	0.3991	7	0.1550	8	0.8026	8	0.0342	9	0.1362	9	86.80%	11	77	9
GJR-N	0.9685	4	173.7340	7	9.2787	5	0.3878	3	0.1471	4	0.7787	5	0.0321	4	0.1286	7	88.70%	4	43	5
GJR - St	0.9204	2	148.6420	2	8.7418	2	0.3883	4	0.1428	2	0.7364	2	0.0308	2	0.1026	1	90.10%	1	18	2
GJR - GED	0.8729	1	146.1900	1	8.6625	1	0.3671	1	0.1395	1	0.7304	1	0.0293	1	0.1130	2	89.50%	2	11	1
EGARCH-N	1.2628	12	275.1420	12	11.3735	12	0.4189	11	0.1685	12	0.9046	12	0.0361	11	0.1579	12	87.40%	9	103	12
EGARCH- St	0.9617	3	169.3140	5	9.2365	3	0.3979	6	0.1464	3	0.7709	3	0.0323	6	0.1132	3	88.50%	5	37	3
EGARCH- GED	1.0732	8	223.6930	11	10.2588	10	0.3893	5	0.1551	9	0.8247	9	0.0321	5	0.1341	8	88.80%	3	68	8
APARCH-N	1.0875	10	199.9680	9	9.9860	9	0.4059	8	0.1559	10	0.8289	10	0.0345	10	0.1435	10	86.90%	10	86	10
APARCH-St	1.0349	7	172.8760	6	9.5142	7	0.4075	9	0.1514	7	0.7880	7	0.0333	7	0.1144	5	88.10%	6	61	7
APARCH-GED	0.9780	5	168.7410	4	9.2602	4	0.3843	2	0.1476	5	0.7749	4	0.0316	3	0.1258	6	87.80%	7.5	40.5	4

5.3.3. Ten - step-ahead Forecast Evaluation

The model comparison at the ten-step-ahead forecast horizon is given by Table-7. The model that reveals the best forecasting ability lies again with GJR for all the three distributions as highlighted by all the loss functions given in Table-7. The comparison between the other models is complicated because the results are conflicting. For the normal and GED, the second best model is APARCH while it performs the poorest with student-*t*. On the other hand the performance of EGARCH is better with student-*t* versus normal and GED.

Table-8 shows the distribution comparison. The results favor the use of non-normal densities, since all the symmetric and asymmetric GARCH models provide better forecasting performance with non-normal innovations. However, within non-normal distributions GARCH and EGARCH better perform with student-*t* distribution while GJR and APARCH better perform with GED innovations.

Yet again, overall the preeminent model is GJR with GED innovations as obvious by Table-9. All statistical loss functions except HMSE and R^2 strongly support the use of GJR with GED innovations to forecast the volatility of KSE 100 at the ten-step-ahead forecast horizon. The second best model is also GJR with student-*t* innovations.

Table-10: Fifteen-step-ahead Volatility Forecast (Overall Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	R^2	Rank	Sum Rank	Rank
GARCH-N	1.3057	11	376.8350	10	13.9771	11	0.3333	12	0.1402	11	0.9219	11	0.0248	12	0.1040	11	88.60%	12	101	11
GARCH- St	1.1190	6	280.9210	5	12.2034	5	0.3218	8	0.1294	6	0.8181	5	0.0229	8	0.0771	5	90.10%	8.5	56.5	6
GARCH- GED	1.1796	8	323.5380	8	13.0083	8	0.3170	7	0.1331	8	0.8625	8	0.0230	9	0.0915	9	89.10%	11	76	9
GJR-N	1.0498	5	295.3630	7	12.3591	7	0.3042	2	0.1256	5	0.8256	7	0.0210	4	0.0857	7	91.00%	4.5	48.5	5
GJR - St	1.0055	2	247.6760	2	11.4464	2	0.3074	5	0.1225	2	0.7751	2	0.0207	2	0.0693	1	92.20%	1	19	2
GJR - GED	0.9249	1	238.9310	1	11.3914	1	0.2884	1	0.1178	1	0.7679	1	0.0191	1	0.0736	2	91.60%	2	11	1
EGARCH-N	1.4510	12	500.9240	12	15.1625	12	0.3313	11	0.1481	12	0.9671	12	0.0245	11	0.1103	12	90.10%	8.5	102.5	12
EGARCH- St	1.0252	3	280.0880	4	11.8905	3	0.3083	6	0.1240	3	0.7955	3	0.0213	6	0.0754	3	91.00%	4.5	35.5	3
EGARCH- GED	1.1915	10	395.1090	11	13.4655	10	0.3047	4	0.1340	10	0.8679	9	0.0212	5	0.0904	8	91.30%	3	70	8
APARCH-N	1.1814	9	334.6460	9	13.3475	9	0.3232	9	0.1333	9	0.8873	10	0.0230	10	0.0953	10	89.70%	10	85	10
APARCH-St	1.1224	7	282.2960	6	12.2704	6	0.3240	10	0.1294	7	0.8245	6	0.0226	7	0.0761	4	90.70%	6	59	7
APARCH-GED	1.0362	4	271.1030	3	12.1523	4	0.3043	3	0.1247	4	0.8156	4	0.0209	3	0.0814	6	90.50%	7	38	4

Table-11: Twenty-step-ahead Volatility Forecast (Overall Comparison)

Model	MSE1	Rank	MSE2	Rank	MAE1	Rank	MAPE	Rank	TIC	Rank	MAE2	Rank	R2LOG	Rank	HMSE	Rank	$\bar{R}^2$	Rank	Sum Rank	Rank
GARCH-N	1.3492	11	475.1710	10	16.9441	11	0.2916	11	0.1231	11	0.9739	11	0.0197	11	0.0846	11	91.50%	12	99	11
GARCH- St	1.1340	6	324.8810	4	13.4860	4	0.2771	7	0.1126	6	0.8162	5	0.0186	9	0.0642	4	92.80%	9	54	6
GARCH- GED	1.1946	8	388.0730	8	15.4516	8	0.2772	8	0.1158	8	0.9005	8	0.0183	7	0.0748	8	91.90%	11	74	9
GJR-N	1.0701	5	360.5740	7	14.7528	7	0.2651	3	0.1096	5	0.8620	7	0.0167	2	0.0697	7	93.40%	5.5	48.5	5
GJR - St	1.0495	4	298.5630	2	12.7917	1	0.2688	6	0.1082	4	0.7846	2	0.0171	6	0.0593	1	94.50%	1	27	2
GJR - GED	0.9263	1	273.3120	1	12.8225	2	0.2485	1	0.1019	1	0.7726	1	0.0153	1	0.0606	2	94.00%	2	12	1
EGARCH-N	1.5762	12	692.7570	12	18.8929	12	0.2924	12	0.1333	12	1.0411	12	0.0197	12	0.0920	12	92.90%	8	104	12
EGARCH- St	1.0332	2	330.0540	5	13.6925	5	0.2663	4	0.1076	2	0.8095	3	0.0171	5	0.0622	3	93.70%	4	33	3
EGARCH- GED	1.2566	10	522.0150	11	16.4300	10	0.2674	5	0.1189	10	0.9186	9	0.0171	4	0.0748	9	93.90%	3	71	8
APARCH-N	1.2103	9	410.0270	9	15.8875	9	0.2821	10	0.1166	9	0.9253	10	0.0184	8	0.0783	10	92.50%	10	84	10
APARCH-St	1.1550	7	333.3850	6	13.2211	3	0.2778	9	0.1135	7	0.8099	4	0.0186	10	0.0649	5	93.40%	5.5	56.5	7
APARCH-GED	1.0384	3	310.1140	3	13.7412	6	0.2632	2	0.1079	3	0.8245	6	0.0168	3	0.0674	6	93.10%	7	39	4

5.3.4. Fifteen- step-ahead Forecast Evaluation

The model comparison at fifteen-step-ahead volatility forecast also shows that GJR provides the best forecasting ability for all the three distributions.

The forecasting ability of all the symmetric GARCH and asymmetric GARCH models is better with non- normal densities than with normal densities.

The overall performance of GJR is the best in the model comparison and in the densities comparison.

5.3.5. Twenty- step-ahead Forecast Evaluation

At twenty-step-ahead forecasting, the competing models reveal the same forecasting performance with normal and non-normal densities as at the fifteen-step-ahead forecast horizon. So, similar conclusions may be drawn as at fifteen-step-ahead forecast horizon.

It is conspicuous that the R^2 is higher when using non-normal distributions and is highest when using a student-t distribution at all the forecast horizons. Its value also increases from shorter to longer forecast horizons e.g., the highest value at one-day forecast horizon is 30.50% and is 94.50% at twenty-days forecast horizon. But it does not mean the forecast is inadequate at shorter forecast horizons, as explained by Anderson and Bollerslev (1998) and Klaassen (2002). The primary reason for the low R^2 at shorter forecast horizons is the noise in the observed volatility measure. As discussed in Section 4, this noise can be reduced by taking the sum of squared changes over sub-periods. To give an indication of the magnitude of the effect of this noise reduction on R^2, Anderson and Bollerslev compute the R^2 for a GARCH(1,1) model on daily mark/dollar and yen/dollar exchange rates using a single squared daily changes and using the sum of 288 squared five-minute changes in a day. The R^2 increases and they conclude that GARCH does provide good volatility forecasts despite the low R^2 that is typically obtained using the single squared change. For the purpose of this paper, the argument also explains why the R^2 is higher for the longer horizons than for the shorter horizons; in the return series the noise has been reduced in the twenty-day realized volatility by using twenty instead of one squared returns. Further the R^2 is also the highest for the GJR model with student-t innovations, at all the one-day, five-day, ten-day, fifteen-day and twenty-day forecast horizons.

These results generally recommend that volatility forecasts of the KSE 100 index may be improved by using asymmetric GARCH models with non-normal distributions at both short and long forecast horizons. It is also apparent that the GJR model with GED innovations outperforms the other models, at all the forecast horizons.

6. Conclusion

The essential goal of this paper was to compare the performance of several GARCH-type models (GARCH, EGARCH, GJR and APARCH) in estimating and forecasting the volatility of the KSE 100 index. Such a comparison is carried out by comparing one-day, five- day, ten-day, fifteen-day and twenty-day-ahead volatility forecasts. In addition, all the models are estimated assuming both normal and fat-tailed distributions such as student-*t* and GED for the innovations. The comparison was focused on different aspects: the difference between symmetric and asymmetric GARCH (i.e. GARCH versus EGARCH, GJR and APARCH), and the difference between normal and fat-tailed distributions.

Our results show that traceable improvements can be made when an asymmetric GARCH model is used in estimating volatility of the KSE 100 return series. Generally, according to the statistical loss functions, among the competing models, EGARCH and APARCH fit the series better than GJR models. Also, the symmetric GARCH model provides the poorest results to fit the series. All the models with GED innovations fit the series the best. Overall, on the basis of rank of the sum of the ranks of individual loss functions, EGARCH with GED fits the best.

Overall, the empirical results show that GJR with all the three distributions seems to provide superior forecasting performance at all one-day, five day ten-day, fifteen-day and twenty-day-ahead volatility forecasts horizons according to the statistical loss functions. So, it may be concluded that the asymmetric effect is central to estimating the quadratic effect for forecasting. The symmetric GARCH model performs poorly according to the statistical loss functions, especially at shorter forecast horizons. Moreover, non-normal distributions, generally, provide better out-of-sample results than the normal distribution.

Further, according to the different statistical loss functions that evaluate out-of-sample forecasts, the GJR model with GED innovations seems to provide superior forecast ability at both shorter and longer forecast horizons. So, it may be concluded that it is the best way to forecast volatility of KSE 100 index is at shorter and longer forecast horizons.

Appendix

Table-A: Model Selection

Models	LogL	AIC	BIC
ARMA(9, 0)-GARCH(1, 1)	-1722.870	3.3388	3.3627
ARMA(2, 2)-GARCH(1, 1)	-1731.888	3.3395	3.3775
ARMA(3, 0)-GARCH(1, 1)	-1749.293	3.3756	3.4041
ARMA(3, 0)-GARCH(2, 2)	-1745.653	3.3724	3.4105

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